Journal of Thermal Analysis, Vol. 47 (1996) 1611-1613

Book Review

Allan D. Kraus and Avram Bar-Cohen: Design and Analysis of Heat Sinks
John Wiley & Sons, New York-Chichester-Brisbane-Toronto-Singapore, 1995

The term heat sink means, in the first place, the extended heat transfer surface of an electronic component, assembly or module to enhance the transfer of excess thermal energy, generated during operation, to a cooling fluid or the ambient air. However, fins and finned arrays constituting the surface extensions may be applied equally for numerous different purposes, such as surface extension in heat exchangers. There, if they happen to be used on the side of the hot fluid, they are preferably called heat sources.

The book consists of two parts of different natures. Chapters 2 through 8 cover the development of an analysis technique for fins and finned arrays, aimed at sizing and optimizing. The remainder of the text is dedicated to a concise selection of the field literature interpreted by the authors. The two parts together contain almost the entire up-to-date knowledge about extended heat transfer surfaces. The text is supplemented by appendices containing basic matrix algebra and the nomenclature applied.

The authentic analysing strategy, apparently developed by A. D. Kraus and co-workers during years of hard work, is based on a model of the individual fin and the parameters describing its thermal state. The model rests on nine limiting assumptions, known as the Murray-Gardner assumptions. The most important of these are the assumption that the fin has a one-dimensional temperature distribution (i.e. no temperature gradient across the fin thickness) and the assumption that the coefficient of heat transfer to the fin is constant and uniform over the entire surface of the fin. The letter a denotes the coordinate at the fin or spine tip and b stands for the coordinate at the fin or spine base. θ is the temperature excess between the fin surface and the surroundings (K) and q is the heat rate in the fin or spine (W). The presented technique operates by relating the following four variables: θ_a, q_a, θ_b and q_b. The solution of a linear, homogeneous, second-order differential equation for $\theta_{(x)}$ (x is the distance from the fin tip) can be used to determine the heat flow in any fin that satisfies the Murray-Gardner assumptions.

The theory of differential equations guarantees two solutions, $\lambda_1(x)$ and $\lambda_2(x)$, that satisfy the initial conditions:

$$\lambda_1(b) = 1, \quad \frac{d\lambda_1(b)}{dx} = 0; \quad \text{and} \quad \lambda_2(b) = 0, \quad \frac{d\lambda_2(b)}{dx} = \frac{1}{k(b)A(b)}$$

Certain continuity conditions must be imposed on k and A, the conductivity of the fin material and the cross-sectional area. If k or A happens to have discontinuities, the fin is called singular. The final solution to the original problem consists in expressing two (dependent) variables in terms of the other two (independent) variables of the four mentioned above (i.e. two heat rates and two temperature excesses).

0368–4466/96/ $ 5.00

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Since the requirements of a given application decide as to the choice between pairs of dependent and independent variables, a linear transformation may be needed from one arrangement of variables to another. The linear transformation is written in matrix form. Here, the vector of the dependent variables is equal to the product of a matrix and the vector of independent variables. The elements of the matrix depend on the choice of dependent and independent variables. Taking four variables two at a time yields six combinations. The interrelations of three pairs of the six matrices are given in the text. These interrelations are general in the technique suggested by the authors and provide useful flexibility in meeting the most different tasks.

Kraus *et al.* (1978) developed two completely new parameterizations. One of them, called the input admittance (Y_{in}), pertains to regular fins. The other, the thermal transmission ratio (μ), involves singular fins. Singular fins and spines require a special treatment. The definitions of the two parameters appear formally identical:

$$Y_{in} \equiv \frac{q_b}{\theta_b}, \quad \mu \equiv \frac{q_b}{\theta_b}$$

However, in contrast with μ, which can only be applied to a single fin or spine ending in an edge or point, Y_{in} is of general importance in the technique. One of its roles is to substitute the old concept of fin efficiency. The usefulness of fin efficiency is debatable when a finned array is considered. The is because it depends not only on the shape of the fin and the heat transfer conditions, but also on where the fin is mounted in the array. The relation between fin efficiency (η) and input admittance is $Y_{in}=\eta hS$, where h is the heat transfer coefficient and S is the total convective surface in either a single fin or a finned array.

The details of the linear transformations, based on the well-known analytical solutions, are given (Chapter 3) for the following cases: a longitudinal fin with a rectangular profile, the same with one face insulated, a longitudinal fin with a trapezoidal profile, a longitudinal fin with a truncated concave parabolic profile, a radial (cylindrical) fin with a rectangular profile, spines with different uniform cross-sections, a truncated conical spine, and a truncated concave parabolic spine. Solutions for singular fins are given in Chapter 4: a longitudial fin with a triangular profile, with a longitudinal fin with a concave parabolic profile, a conical spine, a concave parabolic spine. To involve the bond resistance between the fin and the prime surface in the analyses, the authors have also formulated the linear transformation for a single series resistance and a single shunt resistance.

The cascade algorithm is presented in Chapter 5. With this procedure in hand, the technique goes beyond the Murray-Gardner assumptions because it allows the change in the heat transfer coefficient along the fin surface to be taken into consideration. The single fin is taken as a series of fins, and the coefficient matrix in the linear transformation can be calculated as a matrix product of the coefficient matrices of the individual fins. The technique is presented by several applications, such as the problem of a longitudinal fin coupled with the bond resistance at its base; fins in a cluster, i.e. one fin attached to the base and several others starting from its tip; a compact heat exchanger stack; and a finned array consisting of curved fins coupled in parallel.

Additional examples and the concept of choking are presented in Chapter 6. The fin is choking the heat flow if increasing the fin height b (the distance between base and tip) decreases the input admittance Y_{in}. If the input admittance pertaining to b=0 is denoted by Y_a and that pertaining to $b \to \infty$ is Y_o, then the relation $Y_o < Y_a$ expresses choking. When $Y_o > Y_a$, the input admittance can be increased by elongating the fin. This permits optimization of the fin as concerns fin height. In the case $Y_o = Y_a$, the value of b has no effect on the heat transfer process.

Finned arrays used in electronic devices and compact heat exchangers often consist of several plate fins of different shapes lumped together along edges. The array is connected by one of more of the fins to the base surface. Application of the method developed for an individual fin to treat an array is a tedious procedure. The authors offer two general methods to facilitate the treatment of finned arrays.

One of them, called node analysis, is applicable to configurations where the finned array does not contain loops in the graph theoretical sense. This method, described in Chapter 7, is based on transformation of the thermal model of the single fin. It is represented as a pi-network consisting of three lumped imaginary fins. When the fins are assembled into an array, the pi-networks are arranged to yield a lumped admittance representation that may then by analysed by network-analyser (matrix) methods. A numerical example of the analysis is shown. The other method, presented in Chapter 8, appears even more general, being applicable to looped fin arrays too. It reduces the analysis of a complicated array, regardless of its size, to a formulation that does not require the generation of the equivalent pi-network required by a nodal analysis.

Chapter 1 contains the basic concepts of heat transfer; the fundamentals of extended surfaces; and the description of thermal resistances in typical chip modules. Chapter 9 is dedicated to optimization of individual fins with the most common shapes. The optimization aims to determine the dimensions that lead to maximum heat transfer for a given fin volume. The procedure followed involves finding the maxima of functions derived from known solutions of the differential equations for the different cases. The last chapter of the book deals with perhaps the most economical, and undoubtedly the most widespread fin geometry, the plate-fin surface extension. It is an array of individual plate fins located at equal distances from each other. The fluid mechanics and heat transfer theory applied specially to plate-fin arrays is presented systematically and in detail. Natural and forced convection heat transfer and also radiation heat transfer are treated with concise explanations. Optimum and nonoptimum fins are analysed through realised applications.

The 407 pages of this book fortify the reader for solving the majority of possible problems in the topic. Ample numerically solved examples help the reader in using the fresh knowledge. Some details of the authors' authentic strategy appear repeated in the text, but this also helps the student in understanding. The Figures yield precise explanations of problems and the notation.

The only reason for a feeling of want might be that no comparison is made with numerical (finite difference, etc.) methods. Numerical methods can by no means overcome the technique presented in the book when individual fins and finned arrays involving one or two connections with the base are treated. On the other hand, numerical methods may come into consideration when the problem is that of a plate-fin array with a common base or the entire electronic package. In the latter cases, the Murray-Gardner assumptions may fail.

The book is of an introductory character and, by its clear explanations, may inspire students to field studies. Nevertheless, it can also be of great benefit to specialists of extended surface design and analysis in a wide range of applications, primarily those of electric and electronic systems and heat exchangers.

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